DAY — 08 SEAT NUMBER | | | | | | |

2025 | VII | 03 | 1100 | J-388 | (E)

MATHEMATICS & STATISTICS (88)
(COMMERCE)

Time : 3 Hrs. (15 Pages) Max. Marks : 80

General Instructions :

(i) All questions are compulsory.

(ii) There are six questions divided into two sections.

(iii) Write answers of Section-I and Section-II in the same answer book.

(iv) Use of logarithmic tables is allowed. Use of calculator is not allowed.

(v) For L.P.P. and Time Series graph paper is not necessary. Only rough sketch of graph is expected.

(vi) Start answer to each question on a new page.

(vii) For each objective type of question (i.e. Q.1 and Q.4) only the first attempt will be considered for evaluation.

SECTION - I

Q. 1. **(A)** Select and write the correct answer of the following multiple choice type of questions (1 mark each) : (6) **[12]**

(i) Matrix $B=\begin{bmatrix} 0 & 3 & 1 \\ -3 & 0 & -4 \\ P & 4 & 0 \end{bmatrix}$ is skew-symmetric then value of P is

(a) 1

(b) -1

(c) 0

(d) -3

(ii) If $y=e^{\log x}$ then $\frac{dy}{dx}=$ _____.

(a) $\frac{1}{x}$

(b) $\frac{1}{2}$

(c) $\frac{e^{\log x}}{x}$

(d) 0

(iii) $\int (1-x)^{-3}dx=$ _____.

(a) $\frac{1}{2}(1-x)^{-2}+c$

(b) $\frac{1}{2}(1+x)^{-2}+c$

(c) $\frac{1}{2}(1-x)^{-2}+\frac{x}{2}+c$

(d) $\frac{1}{2}(1-x)^{-2}-\frac{x}{2}+c$

(iv) If $\int_0^a 3x^2 dx = 8$ then $a =$ ____.

(a) 0 (b) 2

(c) 8 (d) $\frac{4}{3}$

(v) Area of the region bounded by the curve $y = x^2$, the X-axis and the lines $x = 1$ and $x = 3$ is _____.

(a) $\frac{3}{26}$ sq. units (b) 3 sq. units

(c) 26 sq. units (d) $\frac{26}{3}$ sq. units

(vi) The order and degree of $\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^2 = a^x$ are respectively ____

(a) 1, 1 (b) 1, 2

(c) 2, 1 (d) 2, 2

(B) State whether the following statements are true or false (1 mark each) : (3)

(i) Dual of $(p \wedge \sim q) \vee t$ is $(p \vee \sim q) \vee c$

(ii) For $\int \frac{x-1}{(x+1)^3} e^x dx = e^x \cdot f(x) + c$, where $f(x) = (x+1)^2$

(iii) The integrating factor (I.F.) of $\frac{dy}{dx} + y = e^{-x}$ is e^x

(C) Fill in the following blanks (1 mark each) : (3)

(i) Negation of "Some men are animals" is ____.

(ii) If the average revenue is 45 and elasticity of demand is 3 then marginal revenue is ____.

(iii) To find the value of $\int \frac{10x^9 + 10^x \cdot \log 10}{10^x + x^{10}} dx$, the proper substitution is ____.

Q. 2. **(A)** Attempt any TWO of the following questions (3 marks each) : **[14]** (6)

(i) Determine whether the following statement pattern is tautology, contradiction or contingency.
$[(p \wedge q) \vee (\sim p)] \vee [p \wedge (\sim q)]$

(ii) Find $\frac{dy}{dx}$ if $y = (\log x)^x + x^5$

(iii) Find the area of the region bounded by the parabola $y^2 = 4x$ and line $x = 3$.

(B) Attempt any TWO of the following questions (4 marks each) : (8)

(i) Find MPC, MPS, APC and APS, if the expenditure E_c of a person with income I is given as $E_c = (0.0003) I^2 + (0.075) I$ when $I = 1000$.

(ii) Solve the following differential equation :
$x^2 \cdot y \, dx - (x^3 + y^3) \cdot dy = 0$

(iii) Express the following equations in matrix form and solve them by method of reduction :
$x + 2y + z = 8$
$2x + 3y - z = 11$
$3x - y - 2z = 5$

Q. 3. **(A)** Attempt any TWO of the following questions (3 marks each) : **[14]** (6)

(i) Write the converse, inverse and contrapositive of the following statement.

'If a man is bachelor, then he is happy.'

(ii) Find the inverse of $\begin{bmatrix} 3 & 1 & 5 \\ 2 & 7 & 8 \\ 1 & 2 & 5 \end{bmatrix}$ by adjoint method.

(iii) If $x^5 \cdot y^7 = (x+y)^{12}$ then show that $\frac{dy}{dx} = \frac{y}{x}$.

(B) Attempt any ONE of the following questions (4 marks each): (4)

(i) Evaluate : $\int x^2 \cdot e^{3x} dx$

(ii) Evaluate : $\int_1^4 \frac{\sqrt[3]{x+6}}{\sqrt[3]{x+6}+\sqrt[3]{11-x}} dx$

(C) Attempt any ONE of the following questions (Activity) (4 marks each): (4)

(i) Find the values of x, such that $f(x)$ is increasing function

$f(x) = 2x^3 - 15x^2 - 144x - 7$

Solution :

Given : $f(x) = 2x^3 - 15x^2 - 144x - 7$

$\therefore \ f'(x) = 6x^2 - 30x - 144$

Now, $f'(x) > 0$, as $f(x)$ is increasing

$\therefore \ 6x^2 - 30x - 144 > 0$

$\therefore \ x^2 - 5x - 24 > 0$

$\therefore\ (x-8)(x+3)>0$

Case (I) $x-8>0$ and $x+3>0$

$x>8$ and $x>-3$

$\therefore x>$ ☐

Case (II) $x-8<0$ and $x+3<0$

$x<8$ and $x<-3$

$\therefore x<$ ☐

$\therefore\ f(x)=2x^3-15x^2-144x-7$ is increasing if and only if $x\in(-\infty,$ ☐$)$ or $x\in($ ☐$,\infty)$

(ii) Solve the following differential equation, hence find the particular solution when $x=0,\ y=1$

$$y^3-\frac{dy}{dx}=x\frac{dy}{dx}$$

Solution :

$$y^3=x\frac{dy}{dx}+\frac{dy}{dx}$$

$\therefore\quad y^3=(x+1)$ ☐

$\therefore\quad (x+1)\,dy=y^3dx$

Separating the variables, we get

$$\frac{1}{y^3}dy=\frac{1}{x+1}dx$$

Now, integrating, we get

$\therefore\quad \int\frac{1}{y^3}\,dy=\int\frac{1}{x+1}\,dx$

$\therefore\quad -\frac{1}{2y^2}=$ ☐$+c$ ....(I)

which is required general solution

put $x=0$ and $y=1$ in (I)

$$-\frac{1}{2(1)^2} = \log|0+1| + c$$

$\therefore$ $\square = c$

$\therefore$ $-\frac{1}{2y^2} = \square - \frac{1}{2}$

is the particular solution.

SECTION - II

Q. 4. **(A)** Select and write the correct answer of the following multiple choice type of questions (1 mark each) : **[12]** (6)

(i) The sum due is also called as ____

(a) True discount

(b) Face value

(c) Present value

(d) Cash value

(ii) Following are different types of insurance

(I) Life insurance

(II) Health insurance

(III) Liability insurance

(a) Only II

(b) Only III

(c) Only I

(d) All the three

(iii) $\left|b_{xy} + b_{yx}\right| \geq$ ____

(a) $2\ |r|$

(b) $2r$

(c) r

(d) $|r|$

(iv) If $P_{01}(L)=120.4$ and $P_{01}(P)=130.6$ then $P_{01}(D-B)$ is ____

(a) 25.1

(b) 60.2

(c) 125.5

(d) 65.3

(v) $F(x)$ is c.d.f. of discrete r.v. X whose distribution is

X_i	–2	–1	0	1	2
P_i	0.2	0.3	0.15	0.25	0.1

then $F(-3)$ = ____

(a) 1

(b) 0.2

(c) 0.15

(d) 0

(vi) Given p.d.f. of a continuous r.v. X as

$f(x)=\frac{x^2}{3}$, for $-1<x<2$

$=0$, otherwise

then $F(1)$ = ____

(a) $\frac{3}{9}$

(b) $\frac{4}{9}$

(c) $\frac{1}{9}$

(d) $\frac{2}{9}$

(B) State whether the following statements are true or false (1 mark each) : (3)

(i) The banker's discount is also called as commercial discount.

(ii) The optimum value of the objective function of L.P.P. occurs at the centre of the feasible region.

(iii) If $E(X) > \text{Var}(X)$ then X follows Binomial distribution.

(C) Fill in the following blanks (1 mark each) : (3)

(i) The region represented by the inequality $y \leq 0$ lies in ____ quadrants.

(ii) The time required for printing of four books A, B, C and D is 5, 8, 10 and 7 hours. While its data entry requires 7, 4, 3 and 6 hours respectively. The sequence that minimizes total elapsed time is ____.

(iii) If X has Poisson distribution with parameter m and $P(X = 3) = P(X = 4)$ then $m =$ ____.

Q. 5. **(A)** Attempt any TWO of the following questions **[14]**

(3 marks each) : (6)

(i) A person wants to create a fund of ₹ 6, 96, 150 after 4 years at the time of his retirement. He decides to invest a fixed amount at the end of every year in a

bank that offers him interest of 10% p.a. compounded annually. What amount should he invest every year? [Given $(1.1)^4 = 1.4641$]

(ii) Following are given information about advertising expenditure and sales.

	Advertisement expenditure (₹ in lakh) (X)	**Sales (₹ in lakh) (Y)**
Arithmetic mean	10	90
Standard deviation	3	12

Correlation coefficient between X and Y is 0.8

(a) Obtain the regression of Y on X

(b) What is the likely sales when the advertising budget is ₹15 lakh?

(iii) Find y if the cost of living index is 200 for the following data :

Group	**Food**	**Clothing**	**Fuel and Lighting**	**House Rent**	**Miscellaneous**
I	180	120	160	300	200
W	4	5	3	y	2

(B) Attempt any TWO of the following questions (4 marks each) : (8)

(i) A bill of ₹5,475 drawn on 19th January 2015 for 8 months was discounted on 28th February 2015 at 8% p.a. interest. What is the banker's discount? What is the cash value of the bill?

(ii) Following table shows the all India Infant Mortality Rates (per '000) for years 1980 to 1986 :

Years	**1980**	**1981**	**1982**	**1983**	**1984**	**1985**	**1986**
IMR	10	6	5	3	3	1	0

Fit a trend line to the above data by the method of least squares.

(iii) Four new machines M_1, M_2, M_3 and M_4 are to be installed in a machine shop. There are five vacant places A, B, C, D and E available. Because of limited space, machine M_2 cannot be placed at C and M_3 cannot be placed at A. The cost matrix is given below:

Machines	**Places**				
	A	***B***	***C***	***D***	***E***
M_1	4	6	10	5	6
M_2	7	4	–	5	4
M_3	–	6	9	6	2
M_4	9	3	7	2	3

Find the optimal assignment schedule.

Q. 6. **(A)** Attempt any TWO of the following questions (3 marks each) : **[14]** (6)

(i) Obtain 4-yearly centered moving averages for the following time series :

Years	**1981**	**1982**	**1983**	**1984**	**1985**	**1986**
Number of crimes ('000)	40	42	43	42	44	44
Years	**1987**	**1988**	**1989**	**1990**	**1991**	
Number of crimes ('000)	43	46	47	45	46	

(ii) Calculate Walsh's Price Index Number for the following data :

Commodity	**Base Year**		**Current Year**	
	Price	**Quantity**	**Price**	**Quantity**
L	4	8	3	2
M	6	16	8	9
N	8	18	7	32

(iii) A pair of dice is thrown 3 times. If getting a doublet is considered a success, find the probability of two successes.

(B) Attempt any ONE of the following questions (4 marks each) : (4)

(i) The equations of two regression lines are $8x-10y+66=0$ and $40x-18y=214$. Find

(a) The mean values of X and Y

(b) Correlation coefficient between X and Y

(ii) Maximize : $z=60x+50y$

Subject to : $x+2y\leq 40$

$3x+2y\leq 60$

$x\geq 0, y\geq 0$

(C) Attempt any ONE of the following questions (Activity) (4 marks each) : (4)

(i) Find the sequence that minimizes the total elapsed time to complete the following jobs in the order A B. Find the total elapsed time and idle times for both the machines.

Jobs	**I**	**II**	**III**	**IV**	**V**	**VI**	**VII**
Machine A	7	16	19	10	14	15	5
Machine B	12	14	14	10	16	5	7

Solution :

Using the optimal sequence algorithm, the following optimal sequence can be obtained.

□	□	IV	V	III	□	□

Total elapsed time is obtained as follows

Jobs	Machine A		Machine B	
	Time in	Time out	Time in	Time out
□	0	5	5	12
□	5	12	12	24
IV	12	22	24	34
V	22	36	36	52
III	36	55	55	69
□	55	71	71	85
□	71	86	86	91

∴ Total elapsed time T = 91 units

Idle time for machine A = □ units

Idle time for machine B = □ units

(ii) The probability distribution of X is as follows :

x	0	1	2	3	4
$P[X = x]$	0.1	k	$2k$	$2k$	k

Find (a) k (b) $P(X > 2)$ (c) $P(1 < X \leq 4)$

Solution :

The table gives a probability distribution and therefore

$$\Sigma P(X = x) = 1$$

$$P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4) = 1$$

$$0.1 + k + 2k + 2k + k = 1$$

$$6k = 1 - 0.1$$

$$6k = 0.9$$

$$k = \square$$

$$P(X > 2) = P(X = 3) + P(X = 4)$$

$$= 2k + k$$

$$= \square$$

$$P(1 < X \leq 4) = P(X = \square) + P(X = 3) + P(X = 4)$$

$$= \square + 2k + k$$

$$= 0.75$$

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